

# Accelerated Aging Experiments for Prognostics of Damage Growth in Composite Materials

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## ABSTRACT

Composite structures are gaining importance for use in the aerospace industry. Compared to metallic structures their behavior is less well understood. This lack of understanding may pose constraints on their use. One possible way to deal with some of the risks associated with potential failure is to perform in-situ monitoring to detect precursors of failures. Prognostic algorithms can be used to predict impending failures. They require large amounts of training data to build and tune damage model for making useful predictions. One of the key aspects is to get confirmatory feedback from data as damage progresses. These kinds of data are rarely available from actual systems. The next possible resource to collect such data is an accelerated aging platform. To that end this paper describes a fatigue cycling experiment with the goal to stress carbon-carbon composite coupons with various layups. Piezoelectric disc sensors were used to periodically interrogate the system. Analysis showed distinct differences in the signatures of growing failures between data collected at conditions. Periodic X-radiographs were taken to assess the damage ground truth. Results after signal processing showed clear trends of damage growth that were correlated to damage assessed from the X-ray images.

## INTRODUCTION

Use of carbon based composite materials in aerospace structures is increasing due to their superior properties of strength, stiffness, weight, performance, corrosion resistance, etc. to name a few. A dramatic rise is seen in the application of advanced composite materials for aircraft in the last two decades. Current predictions estimate that over the period of next ten years the manufacturing of composites will quadruple at an increasing usage rate of 7% annually [1]. However, due to lack of dependable Structural Health Monitoring (SHM) techniques these systems are currently overdesigned to avoid failures and hence are less cost-efficient.

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Prognostics is defined as estimating the Remaining Useful Life (RUL) of a structure based on a current state assessment, anticipated future load, and environmental conditions. This will allow avoiding catastrophic failures through advance warnings. Augmented with a damage progression model and condition monitoring data, the prognostic algorithm can extrapolate damage growth trajectory and provide an estimate of the RUL [1]. The damage growth model may be physics based or derived from historical data, detailed understanding of the intrinsic material properties, the structure's geometry, loading environment, etc. For composites, some of these factors are not as well understood as for metals. The anisotropic structure of composites is significantly more complex than metallic structures. Any model or theoretical development based on a particular composite material rarely generalizes to other variants. Where there is a barrage of theoretical models for composite failure there is really no consensus. Thus for any new material, significant model adjustments and fresh validations are required before one could use these models with confidence.

The RUL from prognostics estimates can lead to more informed decisions for future actions such as launch/abort decisions, near term repairs, or maintenance scheduling. Data required for studying fault growth and subsequently developing models for prediction algorithms are rarely available from real applications, especially for composites in new applications areas. Therefore, the scientific community relies on customized accelerated ageing experiments to collect detailed run-to-failure data. From prognostics point of view such experiments address several key issues such as (i) allowing collection of relevant failure data in reasonable timeframe, (ii) ability to control various competing stress factors and in-situ measurements for desired parameters, (iii) develop fault growth models and relate model parameters to identified stress factors, and (iv) validation of prognostics and SHM methods.

The analysis presented in this paper builds on current understanding of fault modes in composites. This paper investigates faults in laminated ply composites. Such structures mainly suffer from two damage types: matrix micro-cracks and inter-laminar delamination. When subject to fatigue loading matrix micro-cracks develop in the matrix through the ply thickness direction, creating high stress concentration at the ply interfaces. As more cracks form, an increased interfacial stress leads to initiation of delamination, which then starts to propagate further. Delamination significantly degrades the strength of the structure and is generally the ultimate cause of failure in composite structures. This implies that the two damage modes co-exist, which should be perceivable from the sensor measurements from the controlled experiments and, therefore, motivates this effort.

Several efforts have characterized composite failures due to fatigue; however, most approaches focused on statistically estimating S-N curves by recording the number of cycles to fail under different loads. That is no failure progression data were collected [2]. Many non-destructive inspection techniques are available for hidden damage characterization but most of them require structure disassembly for inspection. SHM, on the other hand, uses a network of sensors attached to the structure that are able to rapidly inspect the structure. Apart from many other techniques, active PZT-sensor networks have been shown to be promising for guided Lamb waves based interrogation of composite structures [3, 4]. A review of existing guided Lamb waves techniques for composite structural health monitoring indicates that the majority of the research conducted to date has focused on damage localization [4-6]. Also these approaches mostly refer to damage detection without isolating a particular damage

type. Other approaches simulate damage by attaching mass, or drilling a through hole into the structure. Some research papers [7, 8] have reported results on the effect of matrix micro-cracks on Lamb wave propagation, in particular how it affects wave velocity, however they did not quantify matrix micro-crack density or develop diagnosis for a matrix micro-cracking. Other papers have examined methods to study delamination effects using lamb waves [6, 9]. Overall, there appears to be little efforts on fatigue damaging a coupon with in-situ damage state estimation or looking for signatures of cracks and delamination separately[10]. This paper reports on run-to-failure experiments where intermittent ground truth and in-situ characteristics are collected. Growth patterns are analyzed for damage types typical of laminated sheet composites.

## EXPERIMENTAL SETUP

The fatigue cycling experiments serve several objectives– (i) ability to collect run-to-failure data with periodic system health data using health monitoring sensors, (ii) ability to collect ground-truth data for the damage to validate measurement data analysis, (iii) accounting variations between samples of same internal structure (layup), and (iv) characterizing variations between sample of different internal structures. Three symmetric layup configurations were chosen to account for the effect of ply orientation: *Layup 1*:  $[0_2/90_4]$ , *Layup 2*:  $[0/90_2/45/-45/90]$ , and *Layup 3*:  $[90_2/45/-45]_2$ . Torayca T700G uni-directional carbon-prepreg material was used for 15.24 cm x 25.4 cm coupons with dogbone geometry and a notch (5.08mm x 19.3mm) to induce stress concentration. Two six-PZT-sensor SMART Layer® from Acellent Technologies, Inc (Figure 1(a)). were attached to the surface of each sample. This configuration allows six actuators and six sensors to monitor wave propagation through the samples, Figure 1(a) shows one such path from actuator 5 to sensor 8 (path 5→8) that will be used as an example throughout this paper.

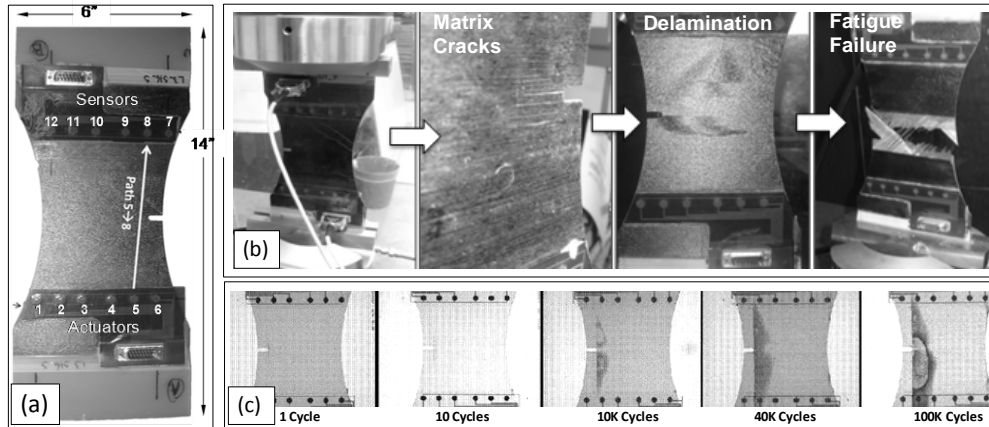


Figure 1(a) Coupon specimen, SMART Layers location, and diagnostic path from actuator 5 to sensor 8. (b) Development of matrix cracks and delamination leading to fatigue failure. (c) Growth in delamination area during the course of fatigue cycling experiment.

Strains of about 0.3-0.4% were estimated at the sensor location. Off-the-shelf data acquisition software and hardware was used to actuate and receive the corresponding signals for the 36 actuator-sensor paths at various actuation frequencies in the range of

150-450 KHz, with an average input voltage of 50 volts and a gain of 20dB. These frequencies were selected so that the fundamental symmetric and anti-symmetric modes can be as distinguishable as possible based on the differences in their phase velocities. Static failure load ( $\sigma_s$ ) was determined through static tests run-to-failure for two or three samples of each layup to determine maximum fatigue load ( $\sigma_f$ ) that was set to 75-85% of  $\sigma_s$ . All tests were performed on an MTS machine with a load ratio (R) of approximately 0.14, following ASTM Standards D3039 and D3479 [11, 12]. The fatigue tests followed a sinusoidal load profile at a frequency of 5Hz. The fatigue cycling tests were stopped every 50,000 cycles to collect PZT sensor data for all paths and interrogation frequencies. X-rays of the samples were taken using a dye-penetrant to enhance X-ray absorption. The main goal of this test procedure is to be able to acquire sensor data as a function of damage progression; Figure 1(c) shows increasing level of damage in the X-ray images.

## DATA ANALYSIS

The approach taken in this project is to understand the damage progression characteristics through experimental run-to-failure data and seek following goals:

- Understand how faults grow in composites under fatigue environments.
- If multiple failure modes co-exist, then how does one isolate and characterize their individual growth characteristics from the monitoring data.
- Identify relevant Condition Indicators (CIs) from the monitoring data.
- Understand the effects of material geometry, construction, and loading sequences.
- Identify and distinguish between various sources of uncertainty in the experimental set up and incorporate them for more accurate predictions.
- Develop empirical models describing fault growths for prognostic modeling.

CIs or features were extracted from monitoring data and the trends observed thereby were compared to those obtained from assessment of X-rays, which is regarded as measured ground truth. This validates the CIs and also helps identify useful features of damage (area, length, intensity, etc.) in the X-rays. Once a good set of CIs is obtained that correlate well with the damage growth observed from the X-rays, an empirical model can be developed for prognostics. X-ray images were processed to extract damage quantifiers like matrix crack density and delamination area. Visible growth in damage was observed for both fault modes (Figure 1(b)). The delamination area grows significantly with fatigue cycling (Figure 1(c)). Delamination areas were measured and plotted against corresponding cycle index. The number of cracks was counted on the path between a sensor-actuator pair and normalized by the path length to obtain an estimate of the crack density. To reduce the uncertainty in the measurements this process was repeated multiple times.

Health monitoring data using Lamb wave propagation in pitch-catch configuration was collected from the PZT sensors to see effects of damage growth in the propagated signal. Separate CIs for matrix cracks and delamination were computed to track the growth of both damage types individually. Since the coupons are relatively small and the velocity of fundamental anti-symmetric  $A_0$  mode is low, it is hard to distinguish the reflected  $A_0$  mode from edges; therefore this work focused only on the fundamental symmetric  $S_0$  mode. In order to distinguish the  $S_0$  mode from the rest of the signal, theoretically calculated group velocity estimates and the known actuator to

sensor path lengths were used to approximate an  $S_0$  mode window as shown in Figure 2(a). Following CIs were computed from the windowed signals.

**Change in Power Spectral Density** - Power Spectral Density (PSD) as a function of time for a given actuation frequency was extracted from Short Time Fourier Transform (STFT) for the signal. The peak value within the specified  $S_0$  mode window decreases as a function of the matrix cracks that developed (see Figure 2(b)). Change in the PSD peak value normalized by the baseline PSD peak was computed. This feature, referred to as the  $\Delta$ PSD throughout this paper, has been shown to correlate well to matrix micro-cracks on any given actuator sensor path [13].

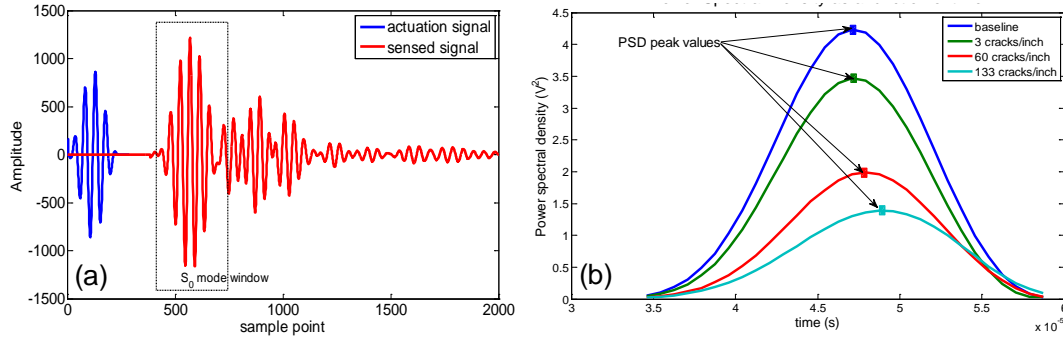


Figure 2(a) Isolating the first  $S_0$  mode by windowing the sensed signal. (b) Change in Power Spectral Density curves with increasing matrix crack density.

**Scatter Energy** - Scatter energy measures wave energy dispersed into the medium along the path due to discontinuities and obstructions. Growth in delamination area increases the scatter proportional to the size of delamination. It is computed as the energy difference for the  $S_0$  mode (Figure 2(a)) between the measured signal and the baseline signal (obtained initially in damage free condition). Normalized scatter energy is calculated as the ratio of scatter energy to baseline signal energy.

**Time of Flight (TOF)** - The Time of Flight (TOF) is the time taken by an actuation signal to reach a sensor, and is a measure of Lamb wave velocity. Physically, delamination degrades the mechanical properties of the coupons, which in turn reduces the Lamb wave velocity leading to increased ToF. The change in TOF is estimated by cross correlating current signal with baseline signal.

## RESULTS AND ANALYSIS

In the search for increasing trends representing damage growth, several features were computed and compared to the trends obtained from X-ray analysis with increasing number of fatigue cycles. The X-ray analysis shows that matrix crack density grows very quickly initially and then flattens out for both layup types (Figure 3 (a)). The cracks grow rapidly again when the loads are ramped up; for instance at cycle 450K when load was increased from 6 to 7 kips for L3S20. The  $\Delta$ PSD feature plotted in Figure 3(b) follows the same matrix crack density growth trends. It is also noticeable how the matrix crack growth difference between layups is captured by the  $\Delta$ PSD.

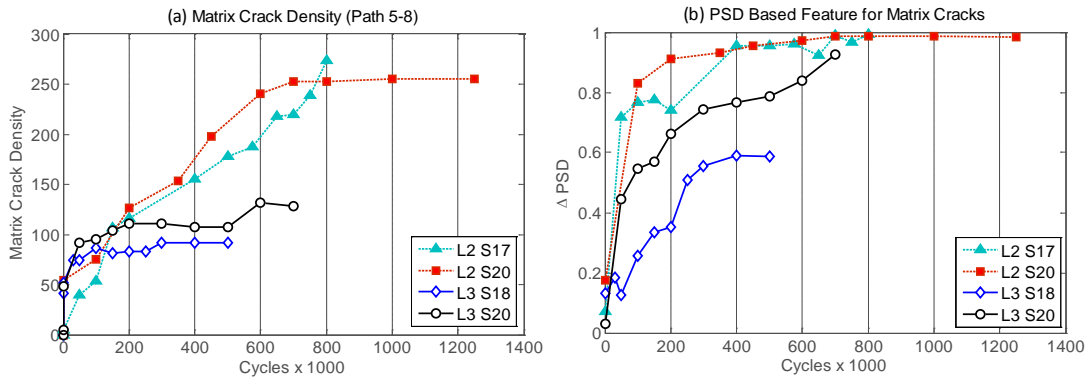


Figure 3. (a) Growth in matrix crack density as observed from X-ray images. (b) PSD based feature to track growth in matrix crack density.

Figure 4 shows corresponding features extracted to track delamination growth. Figure 4(b) shows the normalized scatter energy through cycles for  $L_2$  and  $L_3$  layups. It was observed that the normalized scatter energy for  $L_2$  Layup was relatively higher than that for  $L_3$  layup, which is consistent with the delamination sizes shown in Figure 4(a), but the growth trend is not accurately captured. The change in ToF for  $L_2$  and  $L_3$  layups as a function of cycles is plotted in Figure 4(c). It was observed that the increase in ToF for  $L_3$  Layup was relatively higher than that for the  $L_2$  layup. This is due to the fact that in the case of  $L_3$  layup, stiffness degradation comes from delamination at the 90/45 ply interfaces and matrix cracking in the outer 90° plies. Whereas for  $L_2$  Layup, the overall stiffness degradation is not as significant due to the presence of the 0° outer plies. Despite this mismatch, the monotonically increasing growth trends are observed, for example, L2S17 in Figure 4(a) delamination grows early, then flattens out and as the fatigue loading was ramped up after 600Kcycles, delamination increases significantly. From Figure 4(c) a very similar growth trend is seen in the change for ToF.

Even though these two parameters did not match well individually with the delamination area growth observed from the X-rays, they are promising as signatures of delamination growth trend. A preliminary study on combining these two features indicates that a composite feature such as a product of normalized scatter energy and  $\Delta$ ToF (shown in Figure 4(d)) has well matching trends with delamination area growth. This composite feature shows good correlations to the trends observed in the X-rays for both  $L_2$  samples (L2S17 & L2S20). Likewise for  $L_3$  layups (L3S18 & L3S20) these trends look repeatable, for instance an increase in load at 600K cycles for L3S20 results in increased delamination area, which is also well reflected in the corresponding feature. However, the magnitudes of the delamination features do not correspond to the similar levels for the two layups, i.e. the feature shows similar values for very different magnitudes of delamination area. These differences could be attributed to several reasons that require further investigation: (i) difference in layup types, ( $L_2$ : $[0/90_2/+45/-45/90]_2$  vs.  $L_3$ : $[90_2/+45/-45]_{2s}$ ) and hence effect of delamination geometry and orientation on sensor signals, (ii) errors in the delamination area measurement from the X-ray images, especially if the delamination appears on different interfaces, which is not detectable from X-rays but still affects the signal significantly. Therefore, the layup type should be an important factor in interpreting the results and a good repeatability within a single layup type is desirable. Further studies need to be conducted to assess the accuracy of this composite feature.



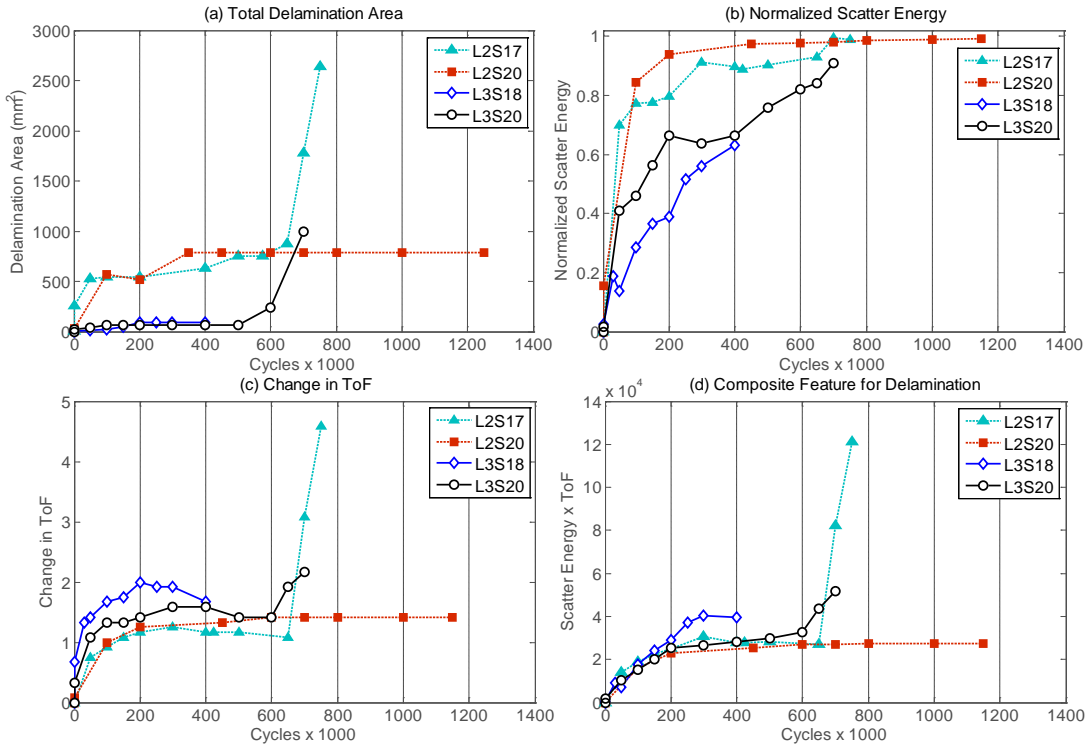


Figure 4. (a) Delamination area as observed from X-ray (b) Normalized scatter energy. (c) Time of flight and (c) Composite feature for delamination.

There were several limitations in the experimental setup that posed technical challenges leading to various uncertainties in the process and are expected to have contributed to some of the differences that were observed above. It is important to consider these sources of uncertainty while interpreting the results from data analysis. Therefore, we present here some such aspects that have been identified and are currently under investigation.

**X-Ray analysis** – (1) The X-ray machine used in this project was analog and resulted in non-uniform digitization leading to variance in contrasts, brightness, scaling, and orientations leading to some uncertainty in ground truth estimation despite calibration steps. (2) X-ray images cannot pinpoint the exact ply interface where the delamination is present. Therefore a single delaminated layer shows same features in the image as for multiple delaminated layers. (3) Matrix crack counting process is a manual process and prone to errors. (4) Cracks appear in different orientations in different layups, and manual counting results in more uncertainties.

**Data Collection Setup** – (1) Wiring connections, and the adhesive all degrade with fatigue cycling limiting our ability to collect high quality fatigue data towards the end of the tests [14, 15]. (2) Since the experiments required the samples to be taken out of the MTS for measurements, re-loading of sample resulted in slight changes in orientation of the coupon that may affect the fault growth as tensile axis changes with orientation. (3) Dye penetrant when wet significantly affected the signal. (4) Manufacturing variability between coupons of the same type also leads to different damage trajectories. (5) Determining optimal load such that coupons break in a

reasonable timeframe has been a challenge. Data on single load levels is not yet available.

## **Prognostic Algorithm Development**

Prognostic algorithm development can take various approaches that may be data-driven or model based. Data-driven approaches learn current damage estimate from condition indicators and damage growth rates from load factors, which then are used to extrapolate the damage to a preset damage threshold to compute estimated RUL. Model based methods make use of a damage progression model instead and extend the current damage estimate through the use of those models. It was determined that so far the collected data is not sufficient to train these models. But with more experiments underway, two individual models for delamination growth and matrix crack density growth will be developed. These models will be used to estimate growth of both damages and then combined to produce a common end-of-life estimate through a recursive Bayesian filtering methods like Particle Filters (PF). PFs have been shown to represent and manage the uncertainty in the prediction process through Importance Sampling, thereby refining the current estimates of multiple damage growth model predictions using evidence from measurement data [16]. Furthermore, a data-driven Gaussian Process Regression approach will also be explored. GPR is a probabilistic technique for nonlinear regression that computes posterior degradation estimates by constraining the prior distribution to fit the available training data [17]. It provides variance around its mean predictions to describe associated uncertainty in the predictions, which will be extremely useful in incorporating the effect of various uncertainties listed above in RUL predictions.

## **CONCLUSIONS & FUTURE WORK**

It was shown in this paper that it is possible to extract separate damage growth indicators that will be useful for prognostic model development. Several features show monotonically increasing trends characterizing damage growth. These indicators were compared to the observations from X-ray images and positive correlations were shown to be found. However, the authors would like to conduct more experiments to establish statistical significance of these results. It is also planned to use strain gauge rosettes at multiple locations to collect additional data in further tests. That will provide additional information about the strain levels during the fatigue tests and help refine data analysis and interpretation. Data analysis, model development, and algorithm work will continue to carry out damage prognosis on composite structures.

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